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BENDING INTERACTION IN SUSPENSION BRIDGES

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STRUCTURAL DIVISION

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BENDING INTERACTION IN SUSPENSION BRIDGES

Haaren A. Miklofsky,[1] A.M. ASCE

SYNOPSIS

This paper describes a direct method of structural design by an interaction diagram as applied to the stiffening trusses of a suspension bridge. The interaction diagram developed gives an overall picture of the effect of changes in dimensions of the cross-section of the stiffening truss upon its stress and deflection. This diagram is drawn by analogy to a bending interaction diagram for a statically determinate structure. The Mount Hope Suspension Bridge is used as an illustrative example.

INTRODUCTION

The cable of a suspension bridge is designed as a string carrying the suspender loads. In long span bridges the stiffening truss is very flexible—simply supported, with a depth to length ratio approximately 1:65—and is not designed as a load carrying member. One function of the stiffening truss, however, is to distribute partial live loads from the deck to the suspenders, so that the vertical movements of the cable from its dead load position are held within specified limits.

Figure 1 is a line diagram of the channel span of the bridge. The shape of the cable under dead load is a parabola. When live load comes on the span, the cable tries to take the shape of a string polygon supporting the live and dead loads, illustrated as that of the unstiffened cable. The truss, tied to the cable by the suspenders, follows its vertical movements, and flexural stresses are set up in the truss which resist the vertical movements of the cable. If the truss were fully flexible—could be bent without resistance—the cable would take the unstiffened position. If the truss were infinitely stiff—could not be bent—the cable would retain its dead load position. Since the truss is neither fully flexible nor infinitely stiff, the cable takes a shape intermediate between these limiting positions.

One problem is to find an economical truss that will limit the cable deflections without overstressing the truss. If trial dimensions are assigned to the truss, methods of analysis are available to find its stresses and deflections.[2] A method of analysis, however, differs from a method of design, in that it will not show directly the effect of a change of dimensions. A change of dimensions of the truss affects both stresses and deflections. What is needed is a picture describing the relationships between stress, deflection, area and depth of the truss.

Such a picture is developed in this paper in the form of a graph, and is called an interaction diagram because a linear change of chord area or depth

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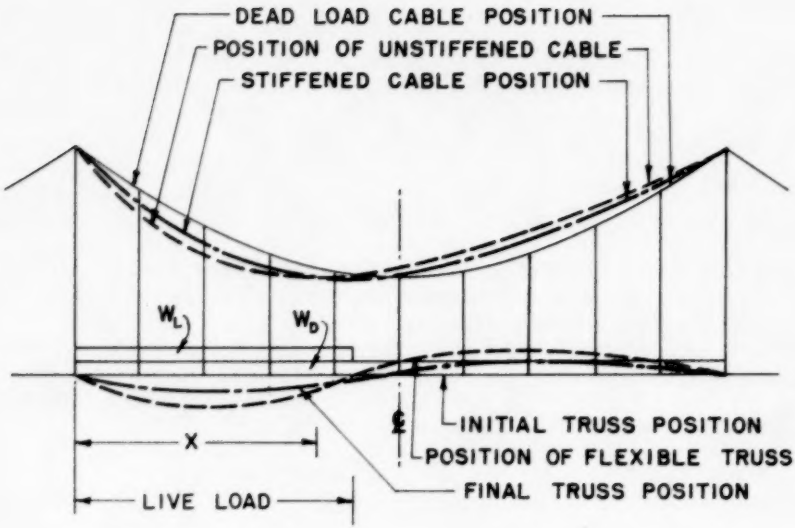
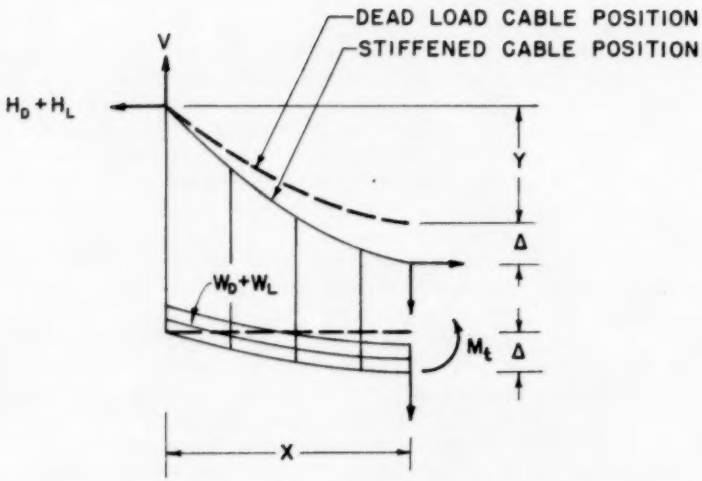
2. See list of references.



(A)



(B)

FIG. 1 DEFLECTIONS OF THE CABLE AND STIFFENING TRUSS UNDER PARTIAL LIVE LOAD.

causes non-linear changes of stress or deflection. The curves for this graph are drawn by analogy to a similar set for a simple beam whose bending action is a result of a combination of transverse bending and axial tension loads. First a study of the stresses and deflections of the beam, as its dimensions are changed, will be presented, and will show the bending interaction due to the two types of loading. Such a study based upon the interpretation of the physical behavior of the statically determinate structure, the beam, aids in understanding the physical behavior of the statically indeterminate structure, the stiffening truss. Later the same type of bending interaction occurring in the stiffening truss due to transverse loading and cable effect will be described, and analogous curves for the stresses and deflections of the truss developed for its variable dimensions.

Bending Interaction Diagram for a Beam: Axial Tension Plus Transverse Bending Loads

Figure 2 shows a simply supported beam subjected to transverse bending loads W and axial tension load P. The center deflection becomes:

$$\Delta = \frac{KM'L^2}{EI} \tag{1}$$

where M' is the bending moment at the center including the direct load P and the transverse loads W. K is a constant dependent upon the distribution of the bending loads and the variation of moment of inertia, I, along the beam.

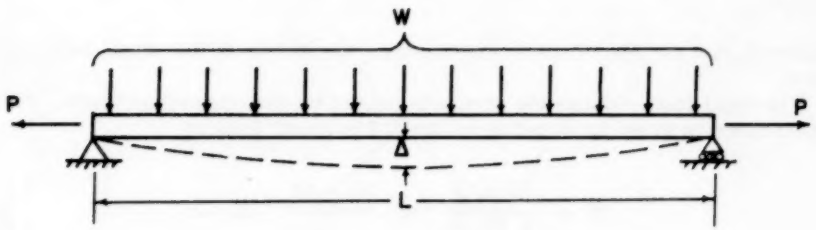


FIG. 2 DIAGRAM OF A SIMPLY SUPPORTED BEAM SUBJECTED TO TRANSVERSE BENDING AND AXIAL TENSION LOADS.

The maximum stress, f, in the outermost fibers also becomes:

$$f = \frac{P}{A} + \frac{Mc}{I} - \frac{P\Delta c}{I} \tag{2}$$

where A represents the cross-sectional area, M the bending moment at the center due to the transverse loads only, and C half the depth. Combining equations (1) and (2):

$$f = \frac{P}{A} + \frac{Mc}{I + \frac{KPL^2}{E}} \tag{3}$$

Now let us consider the second term of equation (3) which is a bending expression for the interaction of two separate bending effects. One is the bending stress induced by the transverse loads alone acting on a simply supported beam. The other is the reduction of bending stress due to the axial load P acting on the sagged beam. From equation (1), M is equal to $M' + P\Delta$. Therefore, the second term of equation (3) can be transposed as follows:

$$\frac{Mc}{I + \frac{KPL^2}{E}} = \frac{Mc}{I} - \frac{P\Delta c}{I} = \frac{Mc}{I + I_p} \tag{4}$$

where I_p can be considered as an additional equivalent moment of inertia resistance of the beam against transverse loading caused by the axial load P.

Now let us show how each separate action affects the stress and deflection of the beam for various depths and areas. It is convenient to vary the depth first, keeping the cross-sectional area constant, and to draw a set of curves of stress and deflection for various depths. Later the cross-sectional area will be changed describing families of curves on the same graph.

Suppose the transverse loads to be the only loads on the beam. The bending stress would then vary inversely with the depth, d, for a constant cross-sectional area. This plots as curve (b) of Figure 3(b), and is represented as follows:

$$f'' = \frac{M\,d/2}{I} = \frac{M\,d/2}{K_1 A d^2} \propto \frac{1}{Ad} \tag{5}$$

where K_1 is a constant dependent upon the distribution of the cross-sectional area.

The axial load reduces the stresses caused by the transverse loads. This effect alone is given by equation (4) when I is zero. Hence:

$$f' = \frac{Md/2}{\frac{KPL^2}{E}} = \frac{Md/2}{I_p} \propto d \tag{6}$$

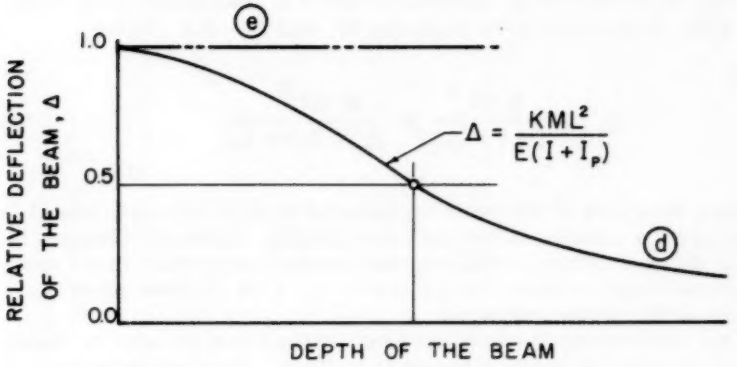
which will plot as line (a) of Figure 3(b).

The reciprocal of equation (4) is equal to the sum of the reciprocals of equations (5) and (6).

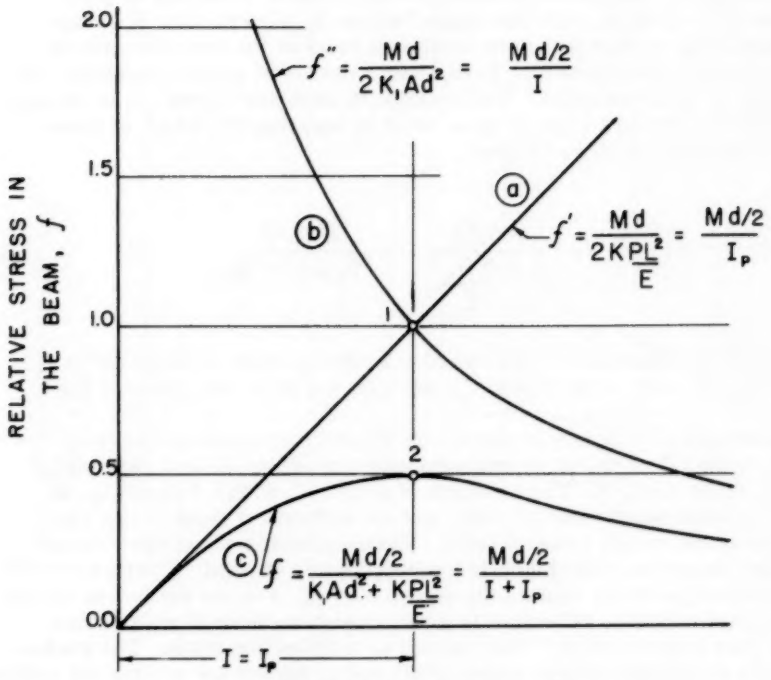
$$\frac{1}{f} = \frac{1}{f''} + \frac{1}{f'} \tag{7}$$

Equation (4) plots as curve (c) on Figure 3(b). This curve is tangent to line (a) when I is negligible in comparison to I_p, and approaches curve (b) as an asymptote when the relative depth of the beam makes I_p negligible in comparison to I. Curve (b) crosses line (a) at the value of depth where I equals I_p; and this is the depth for which curve (c) has its maximum value, point "2", or half the stress value of point "1".

Figure 3(a) shows deflection curve (d) for different depths but with a



(A)



(B)

FIG. 3 DIAGRAM FOR STRESS AND DEFLECTION OF BEAM FOR A VARIABLE DEPTH AND A FIXED CROSS-SECTIONAL AREA.

constant area. It is obtained by equation (1) which is transposed to the more convenient form of equation (8) by replacing M' with M - PΔ. Then:

$$\Delta = \frac{KML^2}{E(I + I_p)} = \frac{KML^2}{E(K_1 Ad^2 + I_p)} \qquad (8)$$

The maximum deflection of the beam represented by line (e) occurs when I is zero, and is given a relative deflection value of unity. Curve (d) reverses in curvature at the depth where I equals I_p and becomes asymptotic to the depth axis as I becomes very large in comparison to I_p. Line (e) determines the vertical scale of the deflection curve.

Curves for different cross-sectional areas of the beam can also be drawn on Figure 3. Now let the depth of the beam be fixed. The position of line (a) is determined by the depth not the area; hence it does not change. The ordinate to curve (b), however, will change inversely with the area. An increase in the area results in an increase in the moment of inertia; hence the intersection of line (a) and curve (b) of Figure 3(b) will be to the left of point "1" at a new value of depth such that again I equals I_p. Curve (c) will change correspondingly so that it has its maximum value at the new intersection depth of line (a) and curve (b). In this way a family of curves for stress can be drawn for different areas. The equation of each new curve (c), is obtained by substituting the new value of area for A in equation (9), which is transposed from equation (4) as follows:

$$f = \frac{Md/2}{I + \frac{KPL^2}{E}} = \frac{Md/2}{K_1 Ad^2 + I_p} \qquad (9)$$

A family of deflection curves can also be drawn from equation (8) by substituting the new value of A in the equation for each new curve of the family.

Figure 4 shows a family of curves for three cross-sectional areas of the beam.[3] Point "1" is the same intersection point of line (a) and curve (b) of Figure 3(b) for area, A. The abscissa of point "1", where I equals I_p, is given a relative depth value of unity; and the ordinate of point "1" is also given a relative stress value of unity. The coordinates of all other stress points are shown in relationship to the coordinates of point "1" which serves as the critical point for scaling the stress curves. For the deflection curves the maximum possible deflection is given a relative value of unity, which occurs when I equals zero.[4] This serves as a deflection scale. The evaluation of the above two critical points is all that is needed for scaling the entire interaction diagram by proportion.

3. Sample calculation: Point 4, Fig. 4. From equation (7), where $f' \propto d$, $f'' \propto \frac{1}{Ad}$ and $\frac{1}{f} \propto \frac{1}{d} + \frac{Ad}{1}$; $\frac{1}{f} = \frac{1}{2} + \frac{2}{1} = 2.50$. Hence $f = 0.40$.

4. Sample calculation: Point 14, Fig. 4. From equation (8) where $\Delta \propto \frac{1}{Ad^2 + 1}$: $\Delta = \frac{1}{1(2)^2 + 1} = 0.20$.

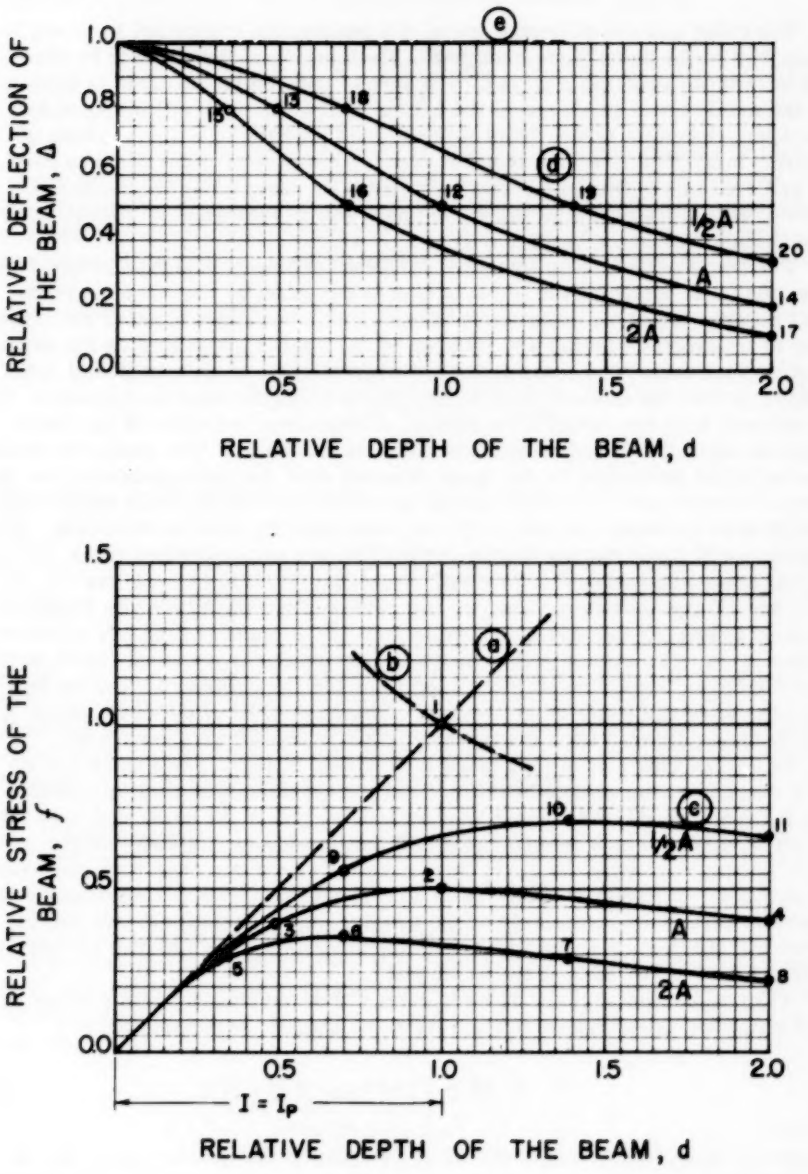


FIG. 4. DIAGRAM FOR PROPORTIONAL LOCATION OF CURVES OF STRESS AND DEFLECTION FOR A VARIABLE DEPTH OR AREA OF THE BEAM.

The Analogy

The cable and the stiffening truss of a suspension bridge act in an analogous way to the beam just described. The function of the cable is to carry the suspender loads as a string. The stress induced in the cable is analogous to the axial stress produced in the beam. When partial live loading comes on the span, the cable moves from its dead load position to a new position of equilibrium. The live load moment causing movement of the cable is partially resisted by the stiffening truss and partially resisted by the deflected cable. Hence the action of the stiffening truss is analogous to the action of the beam considering bending only.

The beam deflection is the same for both the transverse load moment and the axial load moment; the truss is tied to the cable by the suspenders so that it takes the same deflections as the cable. The deflections of the beam due to transverse loading are reduced by the axial load pulling on the sagged beam as a cable. The truss reduces the deflections of an unstiffened cable by taking part of the moment causing deflection from the dead load position, as it deflects with the cable. The amount of deflection reduction of the beam depends upon the distribution of material in that beam. The amount of reduction of cable deflection by the truss depends upon the arrangement of the material in the truss. Curves of stress and deflection of the truss versus any chord area or depth can now be drawn, analogous to those of the beam. The equations of these curves will be derived from a consideration of the statics of the stiffening truss, and they will be similar in form to equations (5), (6), (8), and (9) for the beam. Then an interaction diagram similar to Figure 3 will be drawn for the stress and deflection of the truss versus any depth or chord area. (To simulate the conditions for which the equations were written for the beam, one of the limiting conditions for the cable and truss system is that the span and cable length remain unchanged. The effect of a change of cable length or span upon the interaction diagram will be shown later.)

The cable under dead load takes a parabolic shape. (See Figure 1.) At any section, x, the simple beam moment for the dead load only is balanced by $H_D Y$, where H_D is the dead load horizontal pull of the cable and Y is the vertical distance between the cable and the line joining its ends. When partial live loading comes on the span, the deflection curve becomes a reverse curve with a point of inflection near the middle of the span. Maximum positive deflection occurs approximately at the quarter point for uniform live loading over 0.4 of the span at the same end; and zero deflection for this loading occurs at 0.44 of the span at the same end.[5]

Assume section x at the quarter point.[6] The bending moment induced in the stiffening truss can be written from the statical relationship:

$$M_t = M - (H_D + H_L)(Y + \Delta) \qquad (10)$$

where Δ is the deflection of the cable or truss at the quarter point, M_t is the moment taken by the stiffening truss and M is the simple beam moment at the quarter point.

5. Reference 2, pp. 585, 593.
6. This section is chosen for convenience of comparison of published values of stress and deflection obtained by another method of analysis, and is not necessarily the critical section of the span.

The deflection of the truss is related to its moment as follows:

$$\Delta = \frac{K_o M_t \ell_o^2}{E_t I_t} \tag{11}$$

where E_t is the modulus of elasticity of the truss, I_t is the moment of inertia of the cross-sectional area of the truss at the quarter point, ℓ_o is the length of truss between the end and the inflection point, and K_o is a constant dependent upon the section considered and the loading condition. For maximum stress at the quarter point K_o equals 1/9.1, and ℓ_o equals 0.44 of the span.[7]

Combining equations (10) and (11) to eliminate Δ :

$$M_t = \frac{M - (H_D + H_L) Y}{1 + (H_D + H_L) \dfrac{K_o \ell_o^2}{E_t I_t}} \tag{12}$$

The stress in the stiffening truss of depth d then becomes:

$$f_t = \frac{[M - (H_D + H_L) Y]\, d_t/2}{I_t + (H_D + H_L) \dfrac{K_o \ell_o^2}{E_t}} = \frac{[M - (H_D + H_L) Y]\, d_t/2}{I_t + I_c} \tag{13}$$

where I_c can be considered as an additional equivalent moment of inertia resistance of the truss against transverse loading caused by cable action.

Equation (13) is a bending expression for the interaction of two separate bending stresses. The first stress is that created in the stiffening truss by simple beam action alone and follows the relationship:

$$f_t'' = \frac{[M - (H_D + H_L) Y]\, d_t/2}{I_t} = \frac{[M - (H_D + H_L) Y]\, d_t/2}{K_1 A_t d_t} \propto \frac{1}{A_t d_t} \tag{14}$$

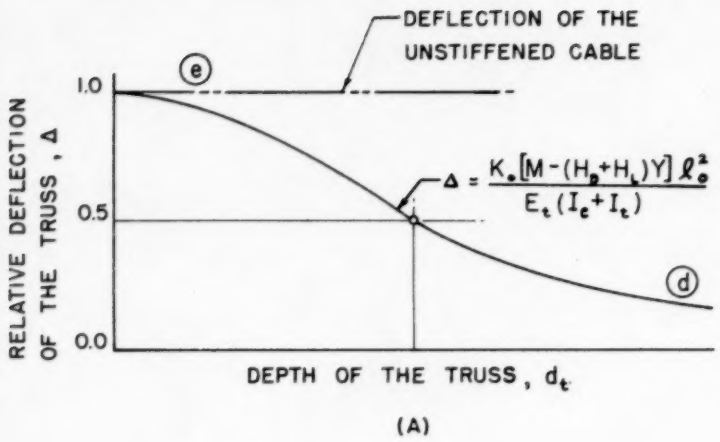
K_1 equals 1/2 when each chord area equals A_t.

The second stress in the truss is that reduction of stress caused by cable action. This effect alone is represented by equation (13) when I_t equals zero, and the truss merely follows an unstiffened cable deflection. The stress then becomes:

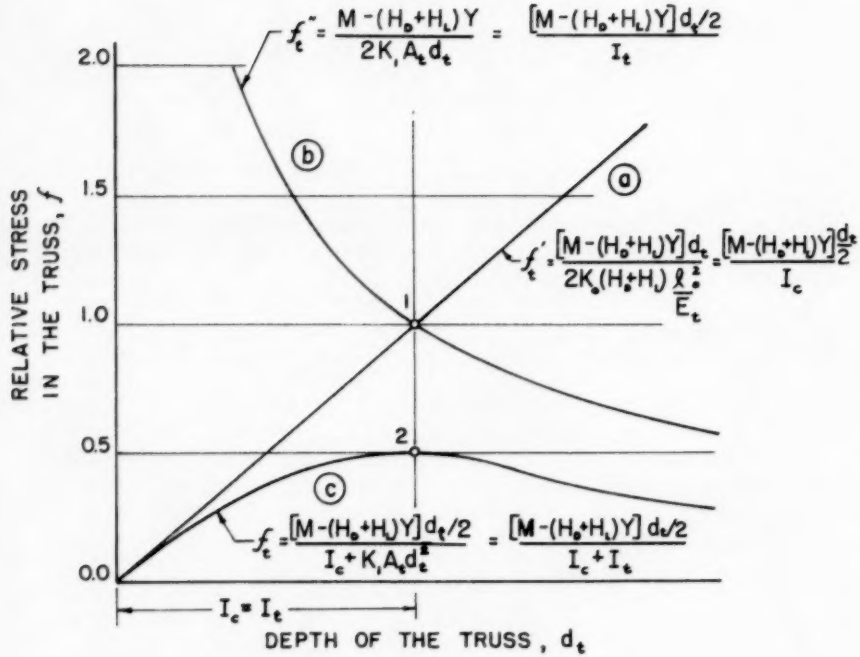
$$f_t' = \frac{[M - (H_D + H_L) Y]\, d_t/2}{(H_D + H_L) \dfrac{K_o \ell_o^2}{E_t}} \propto d_t \tag{15}$$

Equations (13), (14), and (15) for the truss are analogous to equations (9), (5), and (6) for the beam respectively. The reciprocal of equation (13) is equal to the sum of the reciprocals of equations (14) and (15).

7. Discussion of the value of K_o is given in reference 2, p. 593.



(A)



(B)

FIG. 5. DIAGRAM FOR STRESS AND DEFLECTION OF THE STIFFENING TRUSS FOR VARIABLE DEPTH OF THE TRUSS, FOR A CONSTANT CHORD AREA.

From equations (13), (14), and (15), curves of stress in the truss versus the chord area or depth can now be drawn. A variable depth but constant chord area is discussed first. Figure 5(b) shows the curves, (a) through (c), which are identical in form with curves (a) through (c) of Figure 3(b), respectively.

Where line (a) crosses curve (b), $I_t = I_c$. At this intersection depth curve (c) has its maximum value, point "2", equal to half the intersection value of point "1".

The deflection curve of the truss, curve (d) of Figure 5(a), can also be drawn. Combining equations (11) and (12) to eliminate M_t:

$$\Delta = \frac{K_o[M - (H_D+H_L)Y]\ell_o^2}{E_t(I_t + I_c)} \qquad (16)$$

Equation (16) for the truss is analogous to equation (8) for the beam.

A family of curves for different chord areas can also be drawn on the same graph, Figure 5. These curves take the same form as the curves of Figure 4. Point "1" of Figure 5 is analogous to point "1" of Figure 4. Once the scale for point "1" is determined for Figure 5, the rest of the graph follows by proportion, as was done in Figure 4. To determine the scale of point "1", find the depth and stress for which line (a) and curve (b) coincide. The scale of the deflection curves is determined by the deflection of the unstiffened cable, which becomes the maximum possible deflection of the system, represented as line (e) of Figure 5(a).

Sufficient points for sketching a beam of three curves are located on Figure 4. Intermediate curves can be sketched in by eye for preliminary design purposes; or additional points which cannot be plotted on Figure 5 by proportion to Figure 4, can be evaluated from equations (13) or (16).

Illustration

An interaction diagram for stress and deflection versus the chord area and depth of the stiffening truss at the quarter point of the channel span will now be drawn for the Mount Hope Suspension Bridge. Data for the Mount Hope Bridge is given in Figure 6.[8] Although the depth and chord area of the main span stiffening truss as designed are given, it will be assumed that they are unknown and that the interaction diagram is to be constructed for any area or depth of the main span truss. Thus the illustration is one of design rather than analysis. When the diagram is drawn, Figure 7, the actual design can be located on it.

Curves (a), (b), (c), and (d), of Figure 7 can be drawn when the quantity $(H_D + H_L)$ is evaluated for solution of equations (14), (15), and (16). $H_D + H_L$ is the horizontal pull of an unstiffened cable at the tower tops, since equation (15) is the result of letting I_t equal zero in equation (13).

The evaluation of the horizontal pull of an unstiffened cable for partial live loading has already been done by other writers in different ways.[9] The author finds it convenient to use a graph of unstiffened cable deflections for varying ratios of live to dead load published by Hardesty and Wessman,[9(a)] and to

8. Reference 2, p. 600.
9. (a) Reference 2, p. 581.
   (b) Reference 4, p. 214.
   (c) Reference 5, p. 1110.

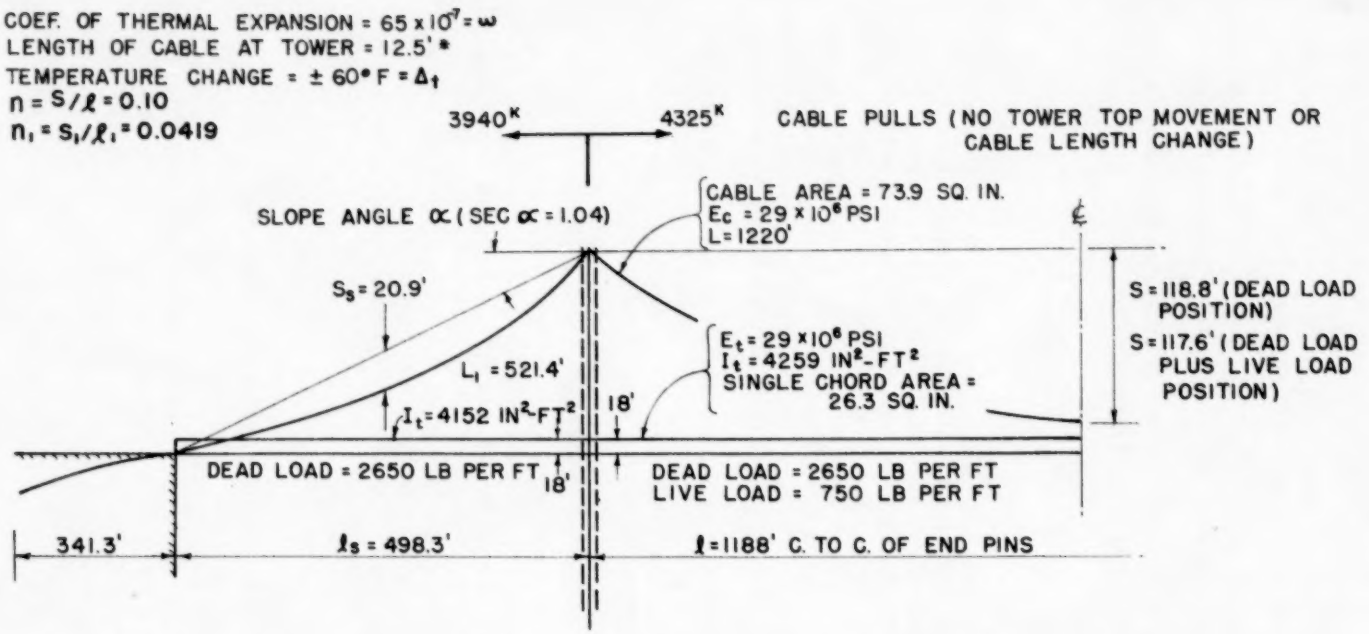

COEF. OF THERMAL EXPANSION = 65 x 10⁻⁷ = ω
LENGTH OF CABLE AT TOWER = 12.5' *
TEMPERATURE CHANGE = ± 60° F = Δt
n = S/ℓ = 0.10
n1 = S1/ℓ1 = 0.0419
3940K
4325K
CABLE PULLS (NO TOWER TOP MOVEMENT OR CABLE LENGTH CHANGE)
SLOPE ANGLE α (SEC α = 1.04)
CABLE AREA = 73.9 SQ. IN.
Ec = 29 x 10⁶ PSI
L = 1220'
℄
Ss = 20.9'
S = 118.8' (DEAD LOAD POSITION)
S = 117.6' (DEAD LOAD PLUS LIVE LOAD POSITION)
Et = 29 x 10⁶ PSI
It = 4259 IN²-FT²
SINGLE CHORD AREA = 26.3 SQ. IN.
L1 = 521.4'
It = 4152 IN²-FT²
18'
DEAD LOAD = 2650 LB PER FT
18'
DEAD LOAD = 2650 LB PER FT
LIVE LOAD = 750 LB PER FT
341.3'
ℓs = 498.3'
ℓ = 1188' C. TO C. OF END PINS

FIG. 6. DATA FOR COMPUTATIONS FOR FIGURE 7 FOR THE MOUNT HOPE SUSPENSION BRIDGE.

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* FOR CABLE LENGTH CHANGES: TOTAL L TAKEN AS THE LENGTH OF MAIN SPAN CABLE PLUS ONE HALF THE LENGTH AT EACH TOWER TOP. TOTAL L_1 TAKEN AS THE LENGTH OF SIDE SPAN CABLE PLUS ANCHORAGE PLUS ONE HALF THE LENGTH AT THE TOWER TOP.

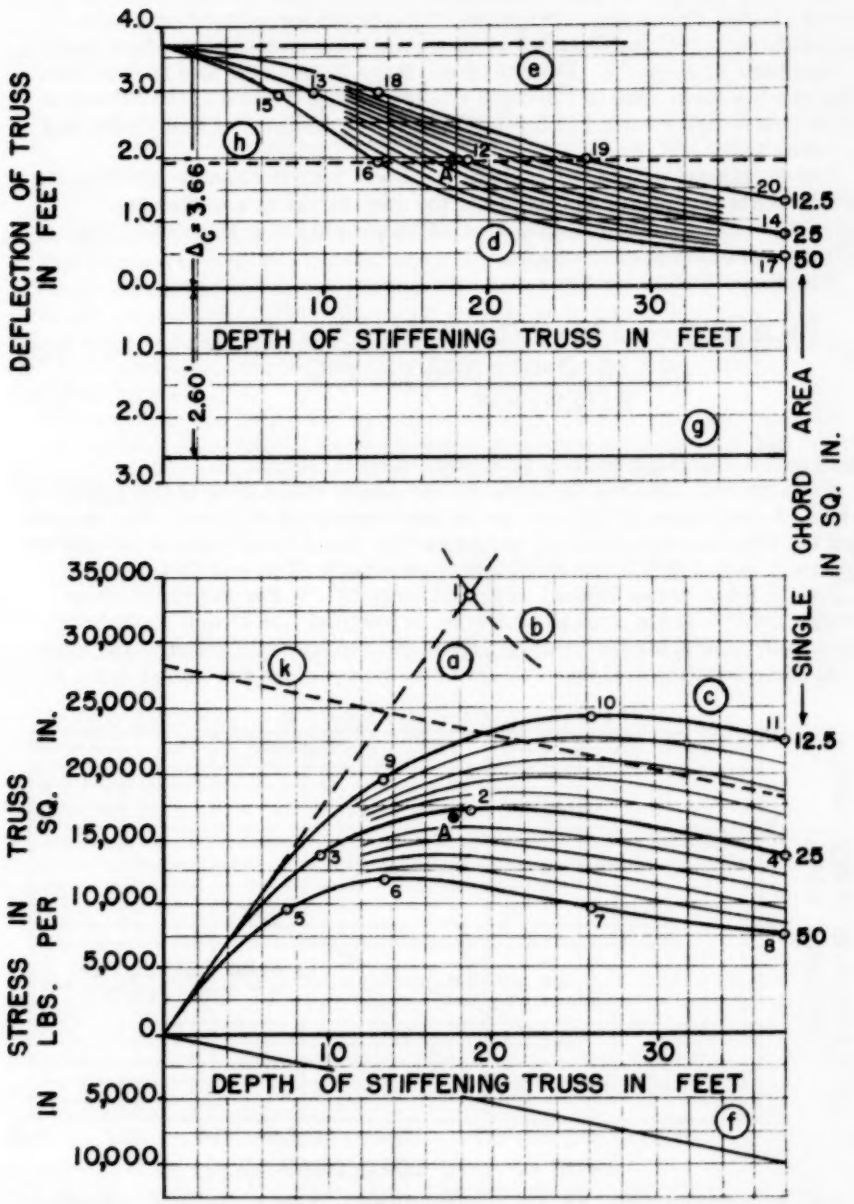

FIG. 7. DIAGRAM FOR STRESS AND DEFLECTION VERSUS DEPTH AND CHORD AREA OF THE STIFFENING TRUSS AT THE QUARTER POINT OF THE MAIN SPAN OF THE MOUNT HOPE SUSPENSION BRIDGE.

calculate the horizontal pull of the cable as the simple beam moment at the center divided by the sag. One curve of the graph for upward deflection at the center for live load over 0.4 of the span at one end full span dead loading is reprinted as Figure 8. For the Mount Hope Bridge, the live to dead load ratio for the main span is 750/2650 = 0.283. From Figure 8, the deflection at the center is 0.0102 x 118.8 = 1.21 feet ↑ (upward). The new center sag becomes 117.6 feet.

The horizontal pull of the cable at the tower tops is found by dividing the simple beam moment at the center of the span by the new center sag. $H_D + H_L$ = 509,900/117.6 = 4336 kips. The simple beam moment at the quarter point is 402,230 foot kips.

Equation (15) then becomes:

$$f_t' = \frac{[402{,}230 - 4336(0.75)118.8]\,10^3(d_t/2)(12)}{\dfrac{4336(0.44 \times 1188 \times 12)^2 10^3}{9.1(29)\,10^6}} = 147.5\,d_t \text{ psi}$$

from which line (a), Figure 7, is drawn. d_t is in inches.

Equation (14) involves the term A_t, the single chord area of the stiffening truss. A trial value of A_t must be chosen to draw the diagram. One suggestion for determining a trial A_t is as follows: Use a trial depth of the stiffening truss as 1-1/2% of the span, and draw a vertical line at that depth on Figure 7, intersecting line (a) at a trial point "1".[10] For the Mount Hope Bridge 1-1/2% of the span is 17.8 feet. A vertical line at this depth intersects line (a) at a stress value of 31,500 lbs per sq. in. Calculate the value of A_t from equation (14) that will give this stress value for a trial depth of

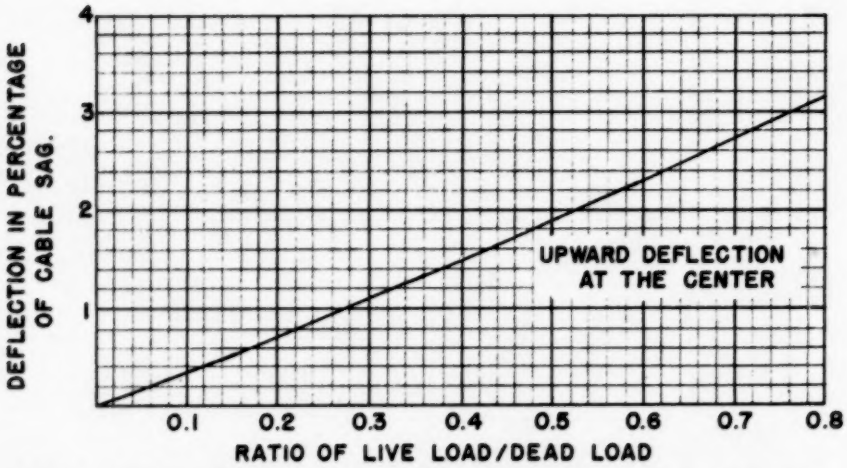

FIG. 8 DEFLECTION OF AN UNSTIFFENED CABLE AT THE CENTER OF THE MAIN SPAN. LIVE LOAD OVER 0.4 OF THE SPAN AT ONE END. DEAD LOAD OVER THE ENTIRE SPAN. CABLE LENGTH AND SPAN UNCHANGED.

10. Many designed bridges have stiffening trusses whose depth is approximately 1-1/2% of its length.

17.8 feet. A_t equals 29.5 sq. in. Since it would be unwise to plot a family of curves for different areas as multiples of 29.5, let the trial value of A_t be 25 sq. in. Now equation (14) is used again for any depth, and becomes:

$$f''_t = \frac{[402{,}230 - 4336(0.75)118.8]10^3(12)}{2(1/2)25\,d_t} = \frac{7{,}627{,}200}{d_t} \text{ psi}$$

from which curve (b), Figure 7, is drawn. d_t is in inches.

The intersection of line (a) and curve (b) locates point "1", and furnishes a scale for its stress and depth. Points "2", "3", "4", etc., of Figure 7 follow by direct proportion to correspondingly numbered points of Figure 4. In this way a sufficient number of points needed to sketch curve (c) for three different chord areas can be plotted. Curves for intermediate areas can be sketched in by eye.

The scale for the deflection curve, curve (d), Figure 7, is found by computing the maximum deflection or the deflection of an unstiffened cable, by letting I_t equal zero in equation (16). Equation (16) then reduces to equation (17) as follows:

$$\Delta = \frac{M - (H_D + H_L)Y}{H_D + H_L} \qquad (17)$$

and the deflection at the quarter for an unstiffened cable becomes:

$$\Delta = \frac{[402{,}230 - 4336(0.75)118.8]10^3(12)}{4336\,(10)^3(12)} = 3.66 \text{ feet}$$

The rest of the curves follow by proportion to Figure 4.

To complete the interaction diagram, the additional stress and deflection due to effect of changes in length of the cable or span must be added to Figure 7.

The Effect of Changes of Cable Length or Span

The deflection of the stiffening truss may be considered the result of two separate deflections, (a) the deflection of the stiffened cable from its dead load curve under unsymmetrical loading, with the cable length and span unchanged, and (b) the deflection of the stiffened cable due to changes in cable length or span. The stress in the truss is considered to be the sum of the stresses due to each deflection.[11]

Three factors contribute to the cable deflection due to cable length or span changes. (1) Live load pull at the tower tops: This pull is due to any condition of live loading which causes an unbalanced force at the tower top in its dead load position. The tower top moves in the direction of the unbalanced force towards a new position of equilibrium. Changes of main span sag are

11. It is not true that each deflection is an independent deflection. Studies by the author indicate that the total deflection changes by about one percent when the effect of each deflection upon the other is considered. Reference 6, p. B-12.

then computed due to the movement of both tower tops. (2) A change of temperature: This causes a change of cable length. With the cable pulls balanced for one position of the tower tops, the sag changes due to the cable length changes can be computed for the main and side spans. Unequal sag changes cause unequal changes of cable pulls at the tower tops in the side and main spans, setting up an unbalanced force which moves the tower tops to a new position of equilibrium. The movement of the tower tops is found. The change of main span sag due to the change of span is then computed, and added to the first sag change to give the total sag change due to a temperature change. (3) Elastic stretch of the cable due to live load stress: This is similar in effect to a temperature change of length, and the procedure of calculation for change of main span sag is the same as for temperature change.

Sag changes due to each of the three factors (live load pull, temperature, and live load stress) can be computed separately. Their total effect, however, is not the result of adding the separate effects, because these factors are interrelated with each other. Any change in main span sag due to temperature causes a change in the cable pull, which in turn causes a change in the elastic stretch of the cable due to live load stress. For any given change of temperature and loading condition, there is only one combination of sag changes due to live load pull, temperature, and live load stress that will satisfy both statics and geometry. That is, the pulls at the tower tops will be in balance, and the original cable length plus any changes of length will equal the final cable length. This combination must be obtained by trial and error. Often the sum of the first approximations are satisfactory for design.

For the maximum positive moment in the stiffening truss at the quarter point of the main span, the combination of cable length and span changes are chosen which give the greatest downward deflection of the main cable. (This means a rise of temperature, and live load in the main span only.) The truss can change the deflection of the cable only by the manner in which it distributes partial live loading to the suspenders from the deck. It will not appreciably resist deflections of the cable due to changes of cable length of span.[12] Hence the flexural stress induced in the truss in following the cable deflections due to span or cable length changes will vary directly as the truss depth.

The deflection at the quarter point is 0.73 of the deflection at the center due to span or cable length changes.[13] This quarter point deflection is added as a constant quantity to the deflection diagram of Figure 7, and is the horizontal line below the depth axis, line (g).[14]

The moment in the stiffening truss at the quarter point due to the cable deflection due to changes of cable length and span is given by the equation:[15]

$$M_t = \frac{7.6\, E_t I_t (\Delta S)}{\ell^2} \qquad (18)$$

12. Studies by the author, reference 6, p. B-14, indicate a deflection reduction under two percent.
13. Reference 2, p. 605.
14. See discussion in footnote 17.
15. Derivation of this equation is given in reference 2, p. 597.

where Δs equals the change of main span sag. The stress then becomes:

$$f_1 = \frac{3.8\, E_t d_t\, (\Delta S)}{\ell^2} \qquad (19)$$

from which line (f), Figure 7, is drawn.

The total stress is then represented by the ordinate from line (f) to curve (c), and the total deflection represented by the ordinate from line (g) to curve (d).

Illustration for Effect of Cable Length and Span Changes

The Mount Hope Bridge will be used as the example. Certain relationships are necessary for calculation, as indicated in Figure 9, and were all taken from a previous publication on suspension bridges.[16] Follow the calculations of Figure 9, and notation of Figure 6.

First assume the tower tops fixed in the dead load position. A rise of temperature of 60 degrees causes an increase in both the side and main cables of 0.339 feet and 0.481 feet respectively.

An estimate is next made as to the value of the final live load pull. When the cable pulls are later computed by statics, the value of the live load pull can be revised, and its effect on the total sag changes recomputed. In this case a final value of 270 kips is made for the live load pull. The changes in cable length due to this pull become 0.112 feet and 0.159 feet for the side and main spans respectively. These changes are added to the temperature changes of length. Figure 9 now gives the relationship between a small change in sag and a small change in the length of cable, with the span length constant. Using these equations we find the side and main cables have increased in sag by 2.27 feet and 1.26 feet respectively.

New cable pulls are then computed for the new sags. For the main span, the simple beam moment at the center is divided by the new sag. For the side span, the truss moment is accounted for in calculating the cable pull.[17] The result is an unbalanced force of 1183 kips, tending to move each tower top in towards the center of the main span. A final trial value of 0.60 feet is assumed for each tower movement. If this is the correct movement, then the cable pulls should be in balance when recomputed for the new sags caused by a change in span.

Figure 9 now gives the relationship between a small change in sag and a small change in span with the cable length constant. In the side span α is the angle of inclination of the chord joining the ends of the side span cable with the horizontal (Figure 6). The sag changes caused by a change in span are therefore computed as an upward movement of 2.90 feet for the side span cable, and a downward movement of 2.30 feet for the main span cable. Recomputing the pulls at the tower top now shows the tower top to be in the final

16. Reference 2, pp. 594-596.

17. The truss moment is subtracted from the simple beam moment when the deflected truss is below its dead load position, and added to the simple beam moment when the truss is above its dead load position. In using equation (11), Hardesty and Wessman recommend that K_0 be evaluated as 1/9.4 when considering the center moment in the stiffening truss caused by changes in sag. Reference 2, p. 597.

SIDE SPAN	MAIN SPAN
Temp: $dL_1 = \omega \Delta_t L_1 = 65 \times 10^{-7} \times 60 \times 868.9$ $= 0.339'$	$dL = \omega \Delta_t L = 65 \times 10^{-7} \times 60 \times 1232.5$ $= 0.481'$
Live Load Stress: Assume $H_L = 270^K$	
$dL = \frac{H_L L_1^2}{A_c E_c \ell_1} = \frac{270 \times 10^3 \times 868.9^2}{73.9 \times 29 \times 10^6 \times 845.8}$ $= 0.112'$	$dL = \frac{H_L L^2}{A_c E_c \ell} = \frac{270 \times 10^3 \times 1232.5^2}{73.9 \times 29 \times 10^6 \times 1200.5}$ $= 0.159'$
Total $dL_1 = 0.339 + 0.112 = 0.451'$	Total $dL = 0.481 + 0.159 = 0.640'$
$ds_1 = \frac{3 \sec^3 \alpha}{16(n_1)} dL_1 = \frac{3(1.04)^3 \times 0.451}{16(0.0419)}$ $= 2.27' \downarrow$	$ds = \frac{15}{16n(5-24n^2)} dL = \frac{15(0.640)}{16(0.10)(5-24 \times 0.10^2)}$ $= 1.26' \downarrow$
Sag = 20.9' $\quad 3940 \longleftarrow$	$\longrightarrow 4336$ Sag = 117.6'
Simple Beam Moment $= 3940 \times 20.9 = 82,350^{KI}$	New Cable Pull $= \frac{509,900}{117.6+1.26} = 4290^K \longrightarrow$
Truss Moment: From Eq. (11)	$3107^K \longleftarrow$
$M_1 = \frac{E I_1 \Delta}{K \ell_1^2} = \frac{29 \times 10^6 \times 4152 \times 2.27}{\frac{1}{9.4} \times 498.3^2 \times 10^5} = 10,350^{KI}$	Unbalance 1183^K
$72,000^{KI}$	Assume each tower top moves 0.60 feet towards the center of the main span.
New Cable Pull $= \frac{72,000}{20.9+2.27} = 3107^K$	
$ds_1 = \frac{3 \sec^2 \alpha}{16(n_1)} d\ell_1 = \frac{3(1.04)^2 \times 0.60}{16(0.0419)}$ $= 2.90' \uparrow$	$ds = \frac{(15-40n^2+288n^4)}{16n(5-24n^2)} d\ell = \frac{(15-40 \times 0.10^2+288 \times 0.10^4) 2(0.60)}{16 \times 0.10(5-24 \times 0.10^2)}$ $= 2.30' \downarrow$
Simple Beam Moment $82,350^{KI}$	New Cable Pull $= \frac{509,900}{117.6+3.56} = 4208^K$
Truss Moment $= 4560(2.90-2.27) = 2,870^{KI}$	Live Load Pull $= 4208 - 3940 = 268^K$
$85,220^{KI}$	Deflection at the quarter point $= 0.73(3.56) = 2.60' \downarrow$
New Cable Pull $= \frac{85,220}{20.9-0.63} = 4204^K$	

FIG. 9. DEFLECTION OF THE QUARTER POINT OF THE MAIN SPAN DUE TO CHANGES OF CABLE LENGTH AND SPAN FOR THE MOUNT HOPE SUSPENSION BRIDGE, WITH THE STIFFENING TRUSS AS DESIGNED IN THE SIDE SPANS ONLY.

position of equilibrium. Subtracting the dead load cable pull from the final cable pull shows the live load pull to be approximately as estimated.

The cable deflection at the quarter point of the main span due to changes of cable length and span now becomes 0.73 x 3.56 = 2.60 feet ↓ , and is represented by a horizontal line below the depth axis, line (g), of Figure 7.

From equation (19), the stress in the stiffening truss at the quarter point becomes $23.2d_t$, from which line (f), Figure 7, is drawn.[18] d_t is in inches.

Point A of Figure 7 shows the location of the design values of depth and chord area for the Mount Hope Suspension Bridge. The total stress is 21,930 lbs. per sq. in., and the total deflection is 4.49 feet. A published value for the maximum positive moment at the quarter-point by the deflection theory for this bridge is 10,390 foot kips, which when substituted into the flexure formula yields a stress value of 21,950 lbs. per sq. in.[19] The deflection at the quarter-point of the main span is also published as 4.5 feet.

Use of Interaction Diagram in Design

Figure 7 is an interaction diagram. Under unsymmetrical live loading the unstiffened cable has a certain deflection from its dead load position. The stiffening truss reduces that deflection, as it interacts with the cable to reduce the total additional moment due to live load causing the cable deflection from its dead load position. The amount of deflection reduction depends upon the moment taken by the truss as it deflects with the cable, which in turn is dependent upon the truss stiffness. For any area or depth, Figure 7 gives a scale on the amount of reduction.

The interaction may also be considered in terms of stress reduction in the stiffening truss, (although it must be emphasized that the purpose of the truss is to control the deflections of the cable). Line (a) of Figure 7 shows the stress that would be induced in the truss, if it were to have the same deflections as the unstiffened cable, but not to take any part of the total additional moment due to unsymmetrical live loading. When the depth of the stiffening truss is less than the value of depth of point "1", line (a) represents the maximum stress the stiffening truss can possibly have, because its deflection is limited by the deflection of the unstiffened cable. However, the truss and the cable interact. In deflecting with the cable, part of the total additional moment is taken by the truss, which reduces the deflection of the cable (and thus of the system), and the stress in the truss is then represented by curve (c). The amount of stress reduction due to interaction is given by the vertical ordinate between curve (c) and line (a) for any depth less than that of point "1".

When the truss depth is greater than the depth value of point "1", the stress in the truss rather than the deflection of the cable will limit the deflection of the system; and the maximum stress the truss could possibly have would be that represented by curve (b). Interaction, however, exists between the cable and truss, with a resultant reduction of deflection of the system.

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18. It is not strictly true that curves (f) and (g) are straight lines. The main span truss does interact to reduce the movements of the tower tops. If the main span truss as designed is accounted for in calculating the final cable pull in Figure 10, the pull will become 4184 kips instead of 4208 kips. This is not considered significant enough to warrant further refinement of the interaction diagram.
19. Reference 2, p. 600.

With the cable and truss acting together, the stress in the truss is once again represented by curve (c). Now the amount of stress reduction due to interaction is given by the vertical ordinate between curve (c) and curve (b) for any depth greater than that of point "1".

After the interaction diagram is constructed, the most suitable dimensions of the main span stiffening truss must be found that will limit the deflection and stress at the quarter point to certain specified values. Specifications for allowable stress for the silicon steel chords of the stiffening truss are 34,000 lbs. per sq. in. for tension and 28,000 lbs. per sq. in. for compression.[20] The assumption is made that all cross-sectional area is in the chords of the truss.

Draw line (k) parallel to line (f) intersecting the vertical axis at 28,000 p.s.i. Any value of depth or area underneath line (k) satisfies the requirements for stress.

Let us assume that the specification for limiting deflection be the same as the design value of deflection. This equals 4.49 feet at the quarter point of the main span. Draw line (h) parallel to line (g) at a deflection value of 4.49 feet above line (g). Any value of depth or area underneath line (h) satisfies the requirements for deflection.

From Figure 7 (if minimum chord area is the criterion for design)[21] the most economical dimensions for the main span truss when trusses as designed exist in the side spans are a single chord area of 16.5 sq. in. and a depth of 23.5 feet.

In design the values for the side span stiffening truss are unknown, and the problem comes up of deciding what the moment of inertia of the side span stiffening truss will probably be, so that its effect upon the choice of dimensions for the main span can be considered. Since the design of the side span stiffening truss will also be influenced by what the dimensions are for the main span truss, there is no way of designing either the side or the main span truss without assuming a trial dimension for the other. One suggestion for obtaining the trial moment of inertia for the side span stiffening truss is as follows: Use the area and depth represented by point "2" of the interaction diagram for the main span truss as trial dimensions for the side span truss. Obtain a limiting stress curve (k) for the main span, based on this trial stiffness for the side span truss. Determine the most suitable dimensions for the main span truss. Adjust the trial moment of inertia of the side span truss to a value equal to or slightly greater than the moment of inertia of the most suitable main span truss. Repeat the process and find a new value for the most suitable main span truss. Readjust the trial moment of inertia of the side span truss until it has a value equal to or slightly greater than the value of moment of inertia for the most suitable main span truss. The trial moment of inertia for the side span truss will then approximate its design moment of inertia.

Another problem which may occur in design is that of having a suspension bridge that cannot be sufficiently stiffened to reduce an undesirable deflection without overstressing the truss. The interaction diagram shows under what conditions that can happen. A certain amount of deflection will always exist, due to changes of cable length and span, and there is no effective way of reducing these deflections by means of a stiffening truss alone. Studies by the

20. Reference 1, p. 587.

21. Another criterion may be the architectural treatment, limiting the ratio of depth to span.

author show that the moment of inertia of the side span truss can change as much as 100% from its design value and only affect the deflection limit of the main span cable at the quarter point of the Mount Hope Bridge by 4.23%. It is unlikely that the stiffness of the side span truss will be increased above the value needed to carry its maximum moment in order to reduce deflections of the main span cable.

A few solutions to this problem present themselves in another way. (1) The main span may be changed, (2) the live to dead load ratio may be changed, (3) the cable sag ratio may be changed. The interaction diagram may be used to ascertain directly the effect of each of these changes upon the design of the suspension bridge stiffening truss. One of the assumptions of this paper was that the primary member (the cable) was designed to carry given loads. The function of the stiffening truss was to reduce excessive deflections of the cable under unsymmetrical live loading. When the required dimensions of the stiffening truss, however, become excessively high, the dimensions of the primary member can then be changed to reduce the required dimensions of the truss. The discussion on the effect of the above changes of the primary member will be in general terms. Quantitative data is not within the scope of this paper; however, it can readily be obtained by the reader.

If the span is decreased (keeping the live to dead load ratio and the cable sag ratio constant), the cable length change due to temperature is decreased. The live load stress and live load pull are also decreased. Hence the cable sag change due to cable length and span changes decrease. Line (g) of Figure 7 moves up. With the same cable sag ratio, but smaller span, the center sag is less, and results in a smaller deflection of the unstiffened cable. The slope of line (a) decreases correspondingly. With less total live load in the main span, the stress in the stiffening truss as governed by curve (b) decreases, and curve (b) moves down. Curves (c) in turn move down. These changes result in a smaller chord area of the main span truss needed to keep the deflection and stress within allowable limits.

If the dead load is increased, changing the live to dead load ratio (but keeping the span and the cable sag ratio constant), line (g) will not change. A lower live to dead load ratio, however, means less deflection of the unstiffened cable, and results in a smaller slope for line (a). Curve (b) will decrease slightly. Curves (c) move down. Less chord area is needed in the stiffening truss to keep the stress and deflection within the specified limits. A larger cable cross-sectional area, however, is needed because of the increased dead load.

If the cable sag ratio is decreased (keeping the span and the live to dead load ratio unchanged), the deflections of the cable due to unsymmetrical live loading is decreased. This results in a decrease of slope of curve (a). Curve (b) will decrease only slightly because the increase of cable pull will offset the reduction of cable deflection. Hence the value of the live load moment causing deflection of the cable from its dead load position will not change very much. Curves (c) move down. Less single chord area in the stiffening truss is needed to limit the cable deflection to a specified value, but more cross-sectional area for the cable is needed to carry the increased pull.

The interaction diagram can be used to observe the effects of the changes previously discussed. Although quantitative results are not given in this paper, they readily can be obtained directly for any set of design conditions. The interaction diagram shows the whole picture of design at a glance.

This paper does not discuss allowable stress values. However, the interaction diagram can be used to determine where high alloy strength steel is

needed, and where structural grade steel may be sufficient for the stiffening truss chords. For example if medium carbon steel with an allowable compressive stress of 21,000 lbs. per sq. in., and an allowable tensile stress of 26,000 lbs. per sq. in. were to be used for the chords of the truss of the Mount Hope Suspension Bridge, the minimum required chord area would become 34 sq. in. with a depth of 16.2 feet, when stiffening trusses as designed exist in the side spans.[22] Since these dimensions do not seem excessive, structural steel might have been used to advantage in the chords of the main span truss for this bridge.

The interaction diagram also aids in studies of the possible use of other structural materials, with different moduli of elasticity. The use of a lighter material than steel, such as aluminum with its lower working stress, might be justified, if the depth and chord area were not too large.

CONCLUSION

This paper shows how to construct and use an interaction diagram in designing the stiffening trusses of a suspension bridge. The diagram is one of stress and deflection for any depth or chord area. Thus the entire range of design can be visualized in one picture. The method applies equally well to analysis.

This paper shows how the action of a complicated statically determinate structure such as the stiffening truss and cable of a suspension bridge can be more readily understood by analogy to the action of a statically determinate structure. In this case the determinate structure is a beam subjected to transverse bending and axial tension loads. The same type of interaction diagram can be developed for the arch, in which case the analogous determinate structure becomes a beam with transverse bending and axial compression loads.

The interaction diagram is simple to draw, and requires only one critical point for its construction. The formulas used are few in number and are not elaborate.

For the suspension bridge the interaction diagram shows the contribution of various members such as the main span stiffening truss, the side span stiffening truss, and the cable towards the final stress and deflection in the main span truss. It shows the contribution of various factors such as cable stretch from live load stress or from change of temperature, and movement of the tower tops toward the final stress and deflection in the main span truss. It shows whether it is more economical to increase the chord area or depth of the truss. It serves as an aid in studying the probable use of other materials than structural steel for the truss. It serves as an aid in studies on allowable stress and deflection values. It partially answers the question as to the possibility of having a suspension bridge that cannot be sufficiently stiffened to reduce an undesirable deflection without overstressing the truss, by showing what stresses and deflections would exist no matter what size truss were used.

Perhaps the most significant contribution of this method for design is that it shows the designer "what to do" to directly determine the most suitable dimensions. It is not a method of trial and error.

22. Reference 1, p. 587.

ACKNOWLEDGMENT

The concept of structural interaction was described by Professor Hardy Cross in his graduate lectures at Yale University. The author undertook to prove the bending interaction diagram for the suspension bridge for his doctorate thesis, and the proof was made as a flitched beam analogy.[23] Later the author decided to revise the work to a single beam analogy for publication. The author sincerely appreciates the inspired teaching and guidance by Professor Cross.

REFERENCES

1. "Design of Mount Hope Wire-Cable Suspension Bridge," Engineering News-Record, Vol. 100, No. 15, Apr. 12, 1938, pp. 585-587.
2. Hardesty, S. and Wessman, H. E. "Preliminary Design of Suspension Bridges," Transactions—American Society of Civil Engineers, Vol. 104, 1939, pp. 579-634.
3. Jakkula, A. A. "A History of Suspension Bridges in Bibliographical Form," Engineering Experiment Station Series, No. 57, Agricultural and Mechanical College of Texas, 1941.
4. Johnson, J. B., Bryan, C. W. and Turneaure, F. E. "The Theory and Practice of Modern Framed Structures," Part II, Tenth Edition, John Wiley and Sons, New York, 1929.
5. Ling-Hi Tsien. "A Simplified Method of Analyzing Suspension Bridges," Transactions—American Society of Civil Engineers, Vol. 114, 1949, pp. 1109-1158.
6. Miklofsky, H. A. "Studies of Bending Interaction: Suspension Bridges," Dissertation presented to the Faculty of the Yale School of Engineering in candidacy for the degree of Doctor of Engineering, 1950.
7. Priester, G. C. "Application of Trigonometric Series to Cable Stress Analysis in Suspension Bridges," Engineering Research Bulletin 12, University of Michigan.
8. Steinman, D. B. "A Practical Treatise on Suspension Bridges," Second Edition, 299 pages; John Wiley and Sons, New York, 1929.
9. Steinman, D. B. "A Generalized Deflection Theory for Suspension Bridges," Transactions—American Society of Civil Engineers, Vol. 100, 1935, pp. 1133-1234.
10. Timoshenko, S. "The Stiffness of Suspension Bridges," Transactions—American Society of Civil Engineers, Vol. 94, 1930, pp. 377-391.
11. Wessman, Harold E. "Related Functions of Cable and Stiffening Truss in Suspension Bridges," Thesis presented to the University of Illinois in partial fulfillment of the requirements for the degree of Doctor of Philosophy, 1936.

23. Reference 6.

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c. Discussion of several papers, grouped by Divisions.
d. Presented at the Atlanta (Ga.) Convention of the Society in February, 1954.
e. Presented at the Atlantic City (N.J.) Convention in June, 1954.

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